Preferential migration and random mobility in population size distribution of municipalities

Yoshihiro Yamazaki and Kenji Takamura

Department of Physics, Waseda University, Ohkubo, Shinjuku, Tokyo 169-8555, Japan

KEYWORDS: stochastic process, log-normal, power-law, exponent, crossover

Some approaches in statistical physics have been applied to economic and social events.[1,2] Recently, Kobayashi, Kuninaka *et al.* reviewed statistical properties of several social and biological phenomena as "complex systems" from the viewpoint of stochastic process.[3] In their review, population size distribution of municipalities in Japan was treated as one example. They stated that log-normal distribution was considered to be a basic one for the complex systems, which were generally described as a multiplicative random growth given by an equation $x(t+1) - x(t) = \eta(t)x(t)$. Here, $x(t)$ is population size at time t for example, and $\eta(t)$ is the growth rate and a random variable. This property is well known as Gibrat's law.[4]

For the population size distribution, the following feature has been recognized; the major part of the distribution obeys log-normal, but the tail part corresponding to large size exhibits power-law.[5–8] Sasaki, Kuninaka *et al.* classified Japanese municipalities into three types (village, town, and city), and found that villages and cities are fitted with log-normal and power-law distributions, respectively.[9] Their conclusion was that the difference in these distributions originated from existence of the lower population threshold in cities, but not in villages.

Power-law distribution is reproducible by an additional effect to the above multiplicative random growth. Particularly, the following models have been applied to the Japanese case. (i) Random growth model:[10] This model treats the multiplicative random growth with an additive random noise, namely Kesten process, which possesses power-law property.[11] (ii) Migration model: There are two approaches focusing on population migration. (ii-a) Tomita and Hayashi adopted an urn model where preferential attachment in complex network was applied.[12–14] (ii-b) Sasaki, Kuninaka *et al.* proposed another model and took population thresholds into account.[9,15] Depending on existence of the thresholds, the size distributions of villages, towns, and cities have been separately reproduced. It is noted that emergence of power-law due to a threshold has been already reported in fragmentation process.[16,17]

In this short note, we review the model (ii-b) with modification by focusing on preferential migration effect, which is a different viewpoint from refs. 9,15. Now, N sites representing each municipality are prepared. Time evolution of $x_j(t)$, the size of j-th site, is described by

$$x_j(t+1) - x_j(t) = \sum_{k=1}^{N} M_{jk}(t,p) x_k(t) \quad , \tag{1}$$

$(j,k = 1, \cdots, N)$. Components of the matrix $M(t,p)$ are $-\mu_j$ for (j,j), $+\mu_j$ for $(\xi_j(p), j)$, and 0 for others. Here, μ_j is positive and given as a uniform random value between 0 and m (< 1) for j-th site at each time. $\xi_j(p)$ is the site number selected by the following rule; (a) Separate all sites except for the j-th site into two groups A and B in which a site has larger and smaller (or equal) populations than x_j, respectively. (b) Select the group A with the probability $\frac{1}{2}(1+p)$ and B with $\frac{1}{2}(1-p)$, where $-1 \leq p \leq +1$. (c) $\xi_j(p)$ is chosen with equal probability from the group selected in (b). It is noted that for the top site having the largest size, no migration occurs if the group A is selected. And vice versa for the bottom site having the smallest size. The difference from the approach of Sasaki, Kuninaka *et al.* is that we do not introduce any thresholds.

In calculation of eq.(1), we set $N = 100$, and $x_j(0) = 1$ for all j as the initial condition. For each m and p, the results were averaged over 100 samples. 10^5 iterations of eq.(1) were executed, and the averaged distributions became steady. Figure 1 shows the averaged cumulative distribution function of $\{x_j\}$ represented by $F(x)$. These distributions can be fitted as log-normal distribution for $x \lesssim x^*$ and power-law distribution ($\sim x^{-\beta}$) for $x \gtrsim x^*$, except for the top site. Here, x^* shows the crossover size. The variations of β and $F(x^*)$ are roughly depicted in Fig.2 as functions of m and p. It is found that the exponent β decreases by increase of m and p, and that $F(x^*)$ decreases as m increases and p decreases. These monotonic dependencies suggest one to one correspondence between $(\beta, F(x^*))$ and (m, p).

We note that p has the meaning of *preference strength*. For positive p, population in a site has a tendency to move to a larger site. And basically, this model has the following feature: the larger a site is, the larger population in migration is. Therefore, enhancement of preferential migration, which is a similar effect to the preferential attachment in refs. 12-14, is realized, and the power-law region becomes wide. Here, it is found that the size of the top site becomes larger than that estimated from the power-law fitting (see Fig. 1 for reference). This is because there is no larger site than the top site, and population migration is limited to a smaller site. In spite of that, the probability to move a smaller site becomes small for

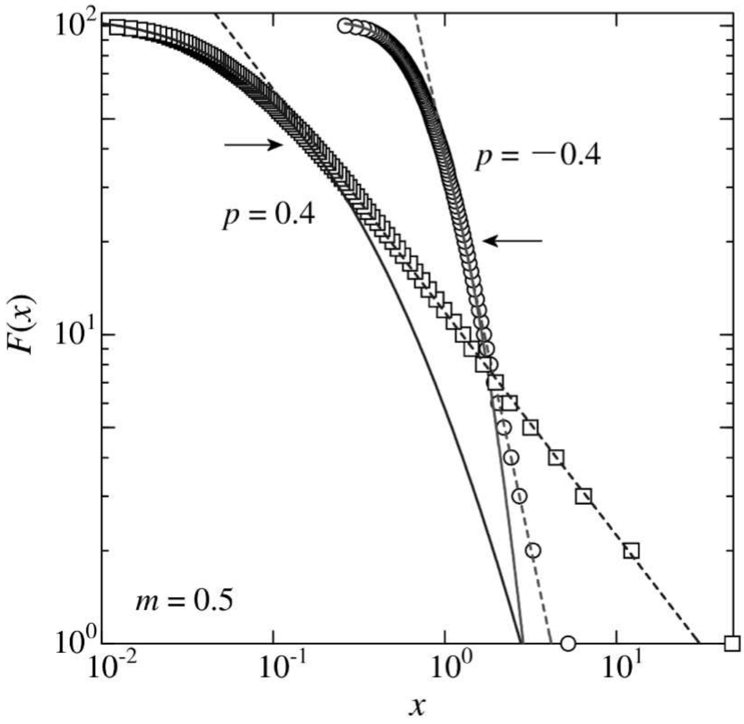
$p > 0$. Then, population migration from the top site hardly occurs. The similar tendency for the top site can be confirmed in the real data: New York in USA (2000)[6)] and Tokyo in Japan (2010).[18)] When p is negative, on the other hand, population in a site tends to move to a smaller site, and the difference in population among the sites becomes small. This effect means averaging of population size. We also note that m determines population *mobility*. High mobility corresponds to large m, namely strong multiplicative noise, which gives rise to domination of log-normality in the distribution. Figure 2 supports this interpretation.

In conclusion, the statistical properties in population size distribution of municipalities can be qualitatively explained by the model (ii-b) without thresholds. The essential point is the two competitive effects: preferential migration and random mobility. The model is able to express not only positive preferential migration, but also negative one, namely averaging migration. And the model gives connection between the statistical values $(\beta, F(x^*))$ obtained from the distribution and the values (m, p) which characterize the above two migration effects in the population dynamics.

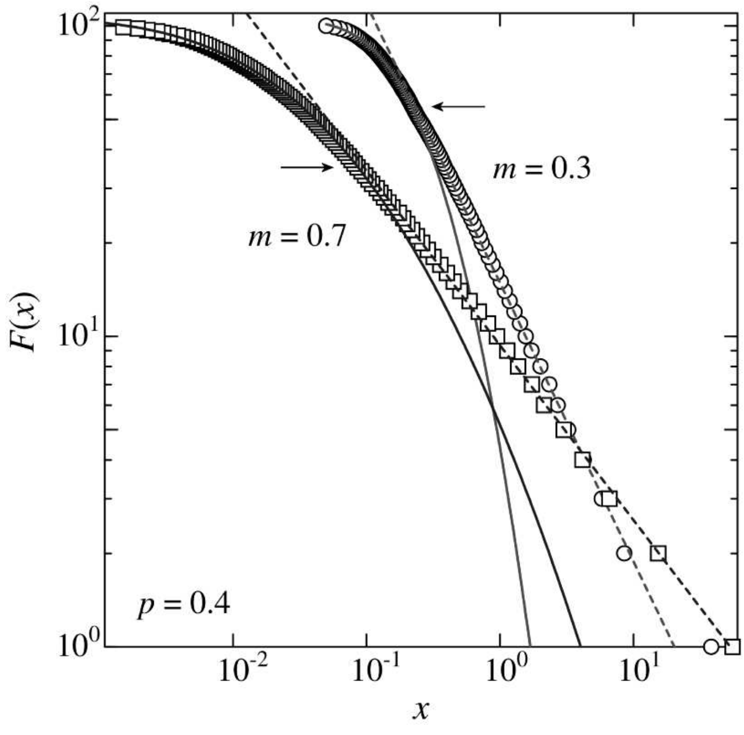
The authors thank Prof. Mitsugu Matsushita for fruitful comments.

References

1) B. K. Chakrabarti, A. Chakraborti, and A. Chatterjee: *Econophysics and sociophysics* (Wiley-VCH, Weinheim, 2006).
2) C. Castellano, S. Fortunato, and V. Loreto: Rev. Mod. Phys. **81** (2009) 591.
3) N. Kobayashi, H. Kuninaka, J. Wakita, and M. Matsushita: J. Phys. Soc. Jpn. **80** (2011) 072001.
4) J. Sutton: Journal of Economic Literature **35** (1997) 40.
5) J. Eeckhout: The American Economic Review, **94** (2004) 1429.
6) M. Levy: The American Economic Review, **99** (2009) 1672.
7) J. Eeckhout: The American Economic Review, **99** (2009) 1676.
8) Z. Fang, J. Wang, B. Liu, and W. Gong: *Handbook of Optimization in Complex Networks* (ed. M. T. Thai and P. M. Pardalos, Springer, 2012) 55.
9) Y. Sasaki, H. Kuninaka, N. Kobayashi, and M. Matsushita: J. Phys. Soc. Jpn. **76** (2007) 074801.
10) S. Tomita and Y. Hayashi: Physica A **387** (2008) 1345.
11) D. Sornette: *Critical Phenomena in Natural Sciences* (Springer Verlag, Berlin, 2004).
12) S. Tomita: Dr. thesis (JAIST, 2008) [in Japanese].
13) J. Ohkubo, M. Yasuda, and K. Tanaka: Physical Review E **72** (2005) 065104.
14) A. -L. Barabási, R. Albert: Science **286** (1999) 509.
15) H. Kuninaka and M. Matsushita: J. Phys. Soc. Jpn. **77** (2008) 114801.
16) M. Matsushita and K. Sumida: Bull. Facul. Sci. Eng. Chuo Univ. **31** (1988) 69.
17) K. Yamamoto and Y. Yamazaki: Physical Review E **85** (2012) 011145.
18) Home page of Statistics Bureau, Ministry of Internal Affairs and Communications, Japan (http://www.stat.go.jp/data/kokusei/2010/index.htm)



(a)



(b)

Fig. 1. (a) p-dependence and (b) m-dependence of the averaged cumulative distribution F as a function of population size x from eq.(1). The arrow near each plot points at $F(x^*)$.

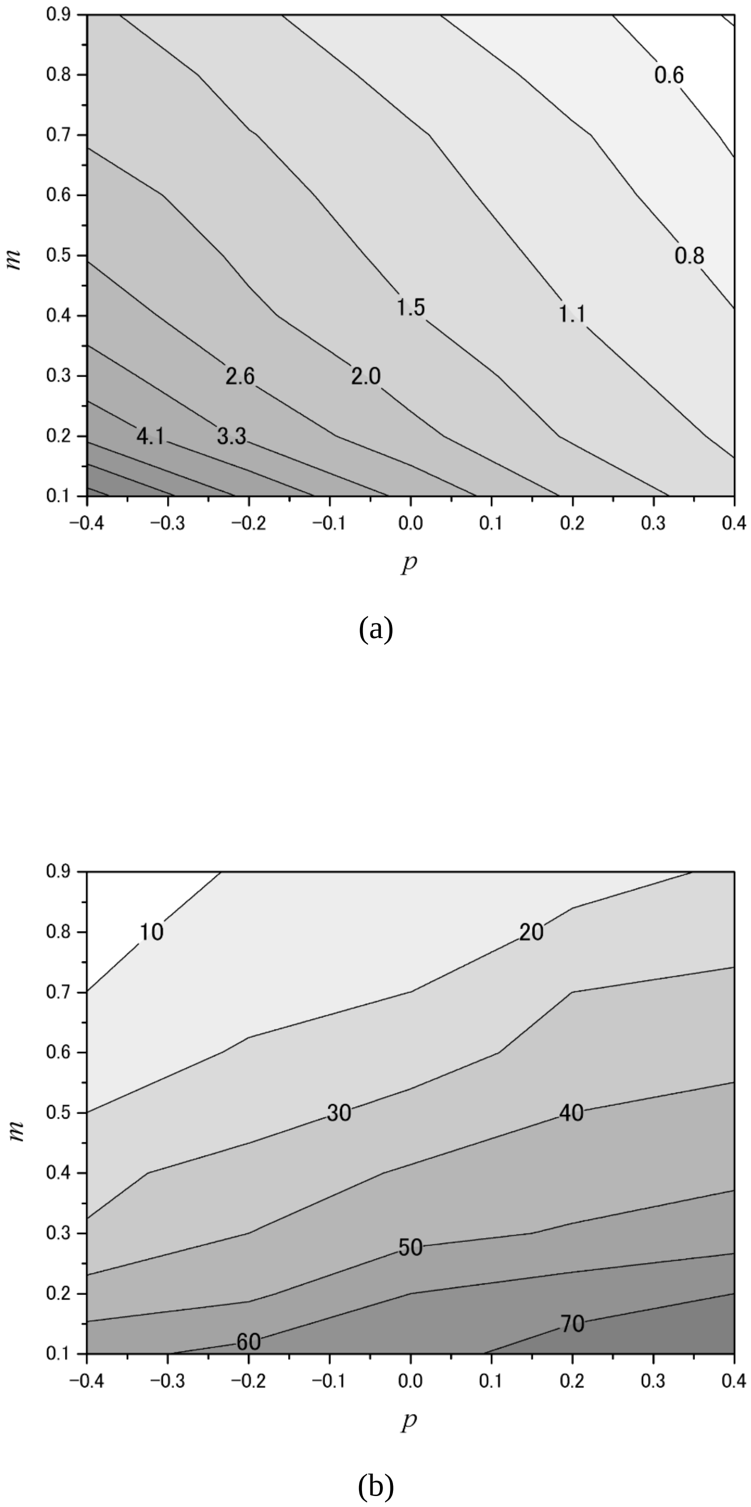


Fig. 2. Contour plots of (a) β and (b) $F(x^*)$ as functions of m and p.