Computing impossible things: the harmonic measure of fractals

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A classic problem in mathematics and physics is to compute the harmonic measure. The harmonic measure may be thought of as the probability distribution for a random walker to land at various points on the surface of an object when launched from outside. For fractals this measure has very interesting scaling properties, but it is notoriously hard to compute numerically because of its huge dynamic range. For example, for a moderate sized DLA cluster the smallest probabilities are of order 10^{-80}.

We have developed a set of numerical methods based on biased sampling which allow us to probe the entire range of the measure. Results will be presented for percolation clusters in 2 and 3 dimensions, Potts model clusters in two dimensions, and DLA in 2 and 3 dimensions. For the percolation and Potts clusters we compare to exact results from conformal field theory. We find excellent agreement.

(Work done in collaboration with D. A. Adams and R. M. Ziff)

Periodic pattern formation in the skin of animals

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Over the past three decades, studies at the molecular level have revealed that a wide range of physiological phenomena are regulated by complex networks of cellular or molecular interactions (*1*). The complexity of such networks gives rise to new problems, however, as the behavior of such systems often defies immediate or intuitive understanding. Mathematical approaches can help to facilitated the understanding of complex systems, and to date these have taken two primary forms. The first of these involves analyzing every element of a network quantitatively and simulating all interactions by computation (*1*). This strategy is effective in relatively simpler systems, for example, the metabolic pathway in a single cell, and is extensively explored in the field of systems biology. However, for more complex system in which spatiotemporal parameters take on importance, it becomes almost impossible to make a meaningful prediction. In such cases, a second strategy involving simple mathematical modeling from which the details of the system are omitted can be more effective in extracting the nature of the complex system (*2*). The reaction-diffusion model (*3*) proposed by Alan Turing is a masterpiece of this sort of mathematical modeling, one that is capable of explaining how spatial patterns develop autonomously.

In the RD model, Turing used a simple system of “two diffusible substances interacting with each other” to represent patterning mechanisms in the embryo, and found that such systems have the ability to generate spatial patterns autonomously. Unfortunately Turing died soon after publishing this legendary paper, but simulation studies of the model have shown that this system can replicate most biological spatial patterns (*4,5,6*). Later, a number of mathematical models (*4*) have been proposed, but in most of them, Turing’s basic idea that “the mutual interaction of elements results in spontaneous pattern formation” is followed. The RD model is now recognized as a standard among mathematical theories that deal with biological pattern formation.

However, this model has yet to gain wide acceptance among experimental biologists. One of the major causes for this is the gap between the mathematical simplicity of the model and the complexity of the real world. The hypothetical molecules in the original RD model have been so idealized for the purposes of mathematical analysis that it seems nearly impossible to adapt the model directly to the complexity of real biological systems. However this is a misunderstanding to which experimental researchers tend to succumb. We can understand the logic of pattern formation using even simple models, and by adapting this logic to very complex biological phenomena, it becomes easier to extract the essence of the underlying mechanisms. Genomic data and new analytic technologies have caused a shift in the target of developmental research from the identification of molecules to understanding the behavior of complex networks, making the reaction-diffusion model more and more important as a tool for theoretical analysis.

In this talk, I summarize our experimental results showing the molecular level mechanism that forms the Turing pattern (skin pigmentation pattern) in zebrafish(4,5), and explain the difference between the simplified mathematical model and complex real mechanism.

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Pattern Evolution – From Phase Separation to Mechanical Fracture

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Phase separation is one of the most fundamental phenomena that create spatially inhomogeneous patterns in materials and nature. It has so far been classified into three types: (i) solid, (ii) fluid, and (iii) viscoelastic phase separation [1,2]. Here we report another phase-separation behaviour accompanying fracture, which is observed under a sufficiently deep quench in polymer solutions [3]. Surprisingly, fracture becomes a dominant coarsening process of the phase separation. Under a deep quench, a transient gel is formed by strong attractive interactions between polymers. The connectivity of the polymer network acts against phase separation and produces the internal stress field. When this stress field exceeds the mechanical stability limit of the transient gel, mechanical fracture takes place: fracture phase separation. The behaviour of viscoelastic and fracture phase separation originates from a strong coupling between composition and deformation field [1]. We demonstrate that the same type of coupling between density and deformation field leads to cavitation of fluid under shear and mechanical fracture of glassy liquid and solid under deformation [4,5]. The key common concept is "dynamic asymmetry". We discuss a common physics underlying these apparently unrelated phenomena and a selection principle of the kinetic pathway of pattern evolution. For example, the only difference between phase separation and fracture may stem from whether deformation is produced internally by phase separation itself or externally by loading.

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Playing with osmotic forces: from mixing to self-propelled swimmers

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When a solute gradient is imposed to a suspension of colloids, particles are observed to drift towards higher or smaller solute concentration. This chemotactic-like motion of particles under concentration gradients of solutes is known as *diffusio-phoresis*. It is an interfacially driven transport phenomenon, which takes its origin in the osmotic driving forces located within a nanometric diffuse interface at the boundary of the particles. However, it is an efficient – and quite unexplored – mean to drive and manipulate particles. In this talk, I will present various experiments exploring the potentialities offered by this phenomenon, from pattern formation, mixing from micro- to macro- scales, to the out-of-equilibrium phase behavior of self-propelled (active) colloidal suspensions.

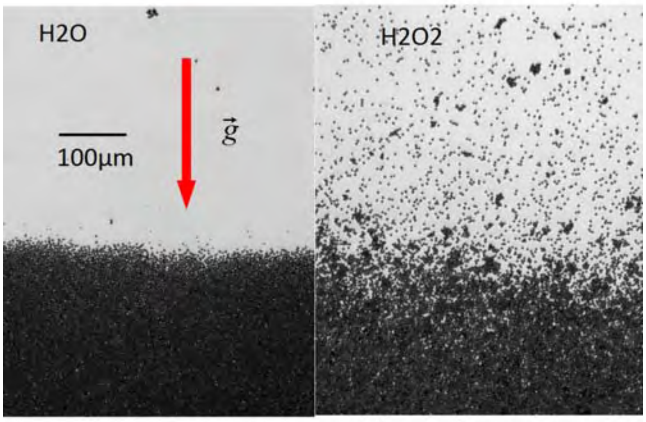

Figure: Particles –made of gold-Platinum janus microspheres do self-propel by consuming a fioul, here the decomposition of hydrogen peroxide. Snapshots of dense suspensions of active suspensions under gravity field: (left) without fioul in bare water; (right) with fioul, showing a hot gas phase, an active solid phase and in between a liquid-like dense phase of clusters.

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Synchronization and Control of Cardiac Systems

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We will introduce our recent theoretical and experimental works in cardiac systems. The first is the synchronization and frequency variation with time in cultured cardiac cells [1], whose oscillatory dynamics can be modelled by coupled excitable elements in the presence of noise. For two such coupled elements, it is found that their frequencies are enhanced by the coupling and will synchronize at a frequency higher than the uncoupled frequencies of each element. As the coupling increases, there is an unexpected peak in the frequency enhancement before reaching synchronization. Similar behaviours are also obtained for a square lattice network of these coupled noisy excitable elements. The simulation results can be understood with a simplified analytic model [2] based on the excitation across a potential barrier whose height is controlled by the coupling. Most importantly, these simulations can quantitatively reproduce the unexpected peak in the variation of the beating rates observed in our experiments [1]. The second is about the dramatic reduction of cardiac alternans by small perturbations in pacing scheme. Predictions and validity of this control method have been verified by both experiments performed with isolated heart preparations and numerical simulations. A nonlinear return map for this novel pacing scheme based on action potential duration restitution response is proposed to explain the working mechanism of the control.

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Pattern Formation in Electro-convection in Liquid Crystals

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Pattern formation in open systems is a very common phenomenon and observed everywhere. A variety of spatial structures is observed, for example, stripe, hexagonal, rectangular and irregular patterns as well as periodic and non-periodic rhythms. Electro-convective patterns in liquid crystals (LCs), which appear at application of electric fields to a thin layer of LCs, especially show such rich aspects. Particularly, due to the initial symmetry of the system, the pattern formation processes are strongly modified. The electro-convection has typically two different initial alignments, the planar and the homeotropic ones. In the former case the initial continuously rotational symmetry is broken against the external fields since the director aligns parallel to the electrodes, while the latter holds the continuously rotational symmetry. This difference leads to completely different routes in the formation processes. For the planar case it shows a successive transition by fully-developed turbulence via regular patterns called the Williams domain (WD), fluctuating WD (FWD: defect turbulence), grid pattern (GP), and dynamic scattering mode (DSM) including others. On the other hand, for the homeotropic case, the irregular flow directly appears from a quiescent state via single supercritical bifurcation. It shows softening of its macroscopic fluctuations near the convective threshold and therefore was named as the soft mode turbulence (SMT). SMT occurs due to nonlinear coupling between the Nambu-Goldstone mode and the short wavelength convective mode, which shows a quite novel spatiotemporal chaos (STC) and rich aspects of complex dynamics.

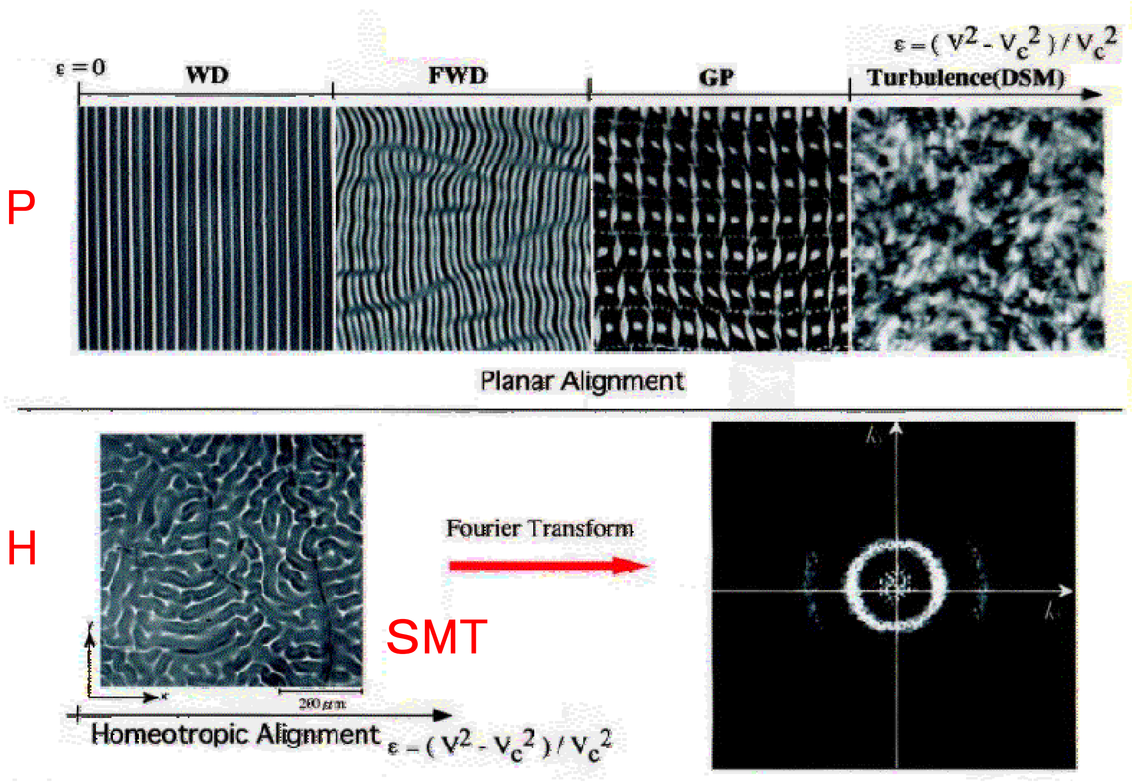

Fig.1. Successive pattern formation in both planar (P) and homeotropic (H) alignments in electro-convection in nematic liquid crystals.

Torahiko's omnivorous courage and high susceptibility to nature

- what modern seismology has lost during its growth -

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Torahiko Terada was one of the initial members who started Earthquake Research Institute of University of Tokyo at 1925/26. The institute was founded after the Great Kanto earthquake of 1923 to enforce seismological research to reduce earthquake damages. Among the members including Prof. Hantaro Nagaoka,a leading physicist at that time, the role of Torahiko seems quite unique. It was the time for initial rapid growing stage of modern seismology with aids of modern instrumentations under the framework of mathematics and physics. He was not so much engaged in this line as a geophysical researcher, while other members pushed forward along. In stead, he seemed to stand in the peripheral regions of seismology, devoting himself to connect seismology with other fields of science. Among over 200 scientific papers he published we can see this in the group of strange papers such as "Colloids and Seismology"[1935],"Analogy between Crack and Electron"[1931] and "Earthquake and Thunderstorm"[1931]. He frequently posed analogies connecting between distant concepts and irrelevant phenomena. Although sometimes they are quite coercive and misdirected his arguments also provided a new standpoint and contributed to widen the view for nature. Because of this style he had been considered as a kind of baseball pitcher throwing moving slow balls instead of straight fast balls and criticized as " a dilettante" scientist.

The reappraisal of Terada's work first came from the fields of nonlinear physics. His style of experimental approaches based on the intuitive analogy might have enchanted present-day researchers. In the field of earth science his approach, sometimes called as an "analog experiment", has been completely replaced by computer simulation and nowadays it is considered out-of-date. But occurrence of the 2011 Tohoku earthquake gave us a chance to reflect on him and his work. The main subject in my talk is about this.

Occurrence of the 2011 Tohoku earthquake has been recognized as a frightening surprise by most seismologists because of its unexpected size[1]. The 1995 Kobe earthquake was also an unexpected blow to seismologists but still its occurrence itself was within the supposition: inland earthquakes with the magnitude 7 are considered to occur as usual crustal activity in Japan. Unexpectedness came from the fact that seismology could not specify the location at Kobe area. The case for the Tohoku earthquake is completely different from that for the Kobe earthquake. Off-Miyagi area had already been a target area for the future earthquake but the estimated size was completely different. This indicates limitations of modern seismology and something essential is still missing as for earthquake generation process. The earthquake started from the point close to the supposed location but it expanded outside, indicating a kind of collective motions of the surrounding faults. The presence/absence of this kind of collaborational/collective movements of group of faults is a key for generation of large earthquake. We know similar behavior in Tonankai area. In the historical development of faults in this area, sometimes near-by faults moved simultaneously to form a mega-earthquake while sometimes they moved separately with time difference of a day to several years. The physics which controls this collective motion is completely unknown at moment.

In the presentation I will review up-to-date understanding of what happened in the 2011 Tohoku earthquake and would like to reflect on why the potentiality of earthquake was overlooked and what is still deficient in modern seismology in relation to Torahiko's research pilgrimage. The story seems a usual case for the example of maturity of one field of science but my focusing point will be that we need such persons like Torahiko who will talk from outside of seismology based on different principles/concepts. Nonlinear physics is a promising partner to work with on this problem and we expect appearance of new Torahiko.

Fracturing Ranked Surfaces

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A “ranked surface” is a lattice on which every site has a rank. Examples are discretized landscapes or sequential percolation. If one cuts the most important connecting bonds a crack appears which has in two dimensions a fractal dimension of 1.217. A classical example is the watershed that separates hydrological basins. In percolation, “bridges” are those sites or bonds which, if occupied, would create the spanning cluster. Suppressing systematically the occupation of these bridges delays the percolation threshold and produces at the end a connected line of bridges which corresponds to the watershed of a random landscape. Also optimal path cracks, the shortest path on loop-less percolation, minimal spanning trees, specific min-max paths and multiple invasion percolation clusters belong to the same universality class. At the percolation threshold bridge percolation exhibits a different exponent, namely ¾, and one finds theta point scaling with a novel crossover exponent. For all dimensions below the upper critical dimension $d_c = 6$ these exponents are calculated. In dimensions larger than two another universality class appears corresponding to the cutting bonds in percolation, i.e. those bonds which if removed would disconnect the spanning cluster.

Quantitative Analysis of Behaviour of Foraging Ants: Decision-making under Conflicting Information

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Foraging behavior of ants is quantitatively investigated focusing on their combined use of visual and chemical cues.

It has widely been known that ants make use of species-dependent several types of cues in their foraging trip, like the polarization of sunlight, landmarks, a class of chemicals generically called pheromone, etc. Especially, by use of visual cues which include the polarization of sunlight and the landmarks, a class of ants are able to come back to their nest taking the almost straight way regardless of generally long and winding paths they took before reaching food.

In the present study, we experimentally study on the selection of the foraging path of garden ants, Lasius Japonicus, that can recognize and response to both visual cues and chemical cues. Specifically, by setting un-optimized initial pheromone path that connects the nest to a feeding site with one corner of finite angles along the paths(Fig.1), we made situations such that the optimal foraging path and along-pheromone path are separated from each other. Hence, with varying the corner angle and the total path length, we observe whether ants keep initial path or newly develop the optimal path.

It was found that, i) for homing ants that got food, a sharp transition from the regime of keeping the initial pheromone path to the regime of developing the optimal path takes place as the relative angle between the direction assigned by the chemical cue and that assigned by the visual cue exceeds a threshold. ii) outbound ants before getting food tend to keep the initial path regardless of the geometry of initial pheromone path. The quantitative data obtained from the video image analysis are shown to discuss the details of the transition mechanism.

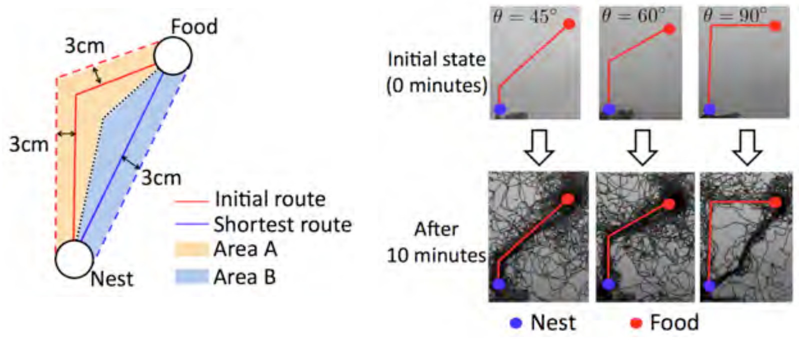

Fig.1 Left: Experimental setup. Right: Initially pasted pheromone path (red lines) and typical trajectories of ants around 10 minutes after starting each experiment.

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Network dynamics in collective motion

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Collective motion patterns are perhaps the most widespread and spectacular manifestations of collective behaviour. The ultimate goal we face is to find unifying principles describing the essential aspects of flocking. A natural approach on the way in this direction is to investigate the delicate dynamics of the interactions between the co-moving individual units. After an introduction to the topic, three new experiments will be discussed. The experimental observations involve the enhanced segregation of two kinds of tissue cells and a study of the hierarchical network dynamics in pigeon flocks as well as their dominance hierarchies. Our animal behaviour studies signal the dawn of a new era of computational ethology.

Fig.1. (Color online) Visualization of the trajectories of the members of a pigeon flock as obtained from downloading the data from the mini GPS devices carried by the birds during their flighs.

Statistical Features of Complex Systems
－Toward establishing sociological physics－

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Complex systems have recently attracted much attention, regardless of natural sciences or sociological sciences [1,2]. Members constituting a complex system evolve through nonlinear interactions among each other. This means that in a complex system the multiplicative experience or, so to speak, history that any member has had produces its present characteristics. We can then anticipate the following. If attention is paid to any statistical property in any complex system, the lognormal distribution is the most natural and appropriate for the standard or "normal" statistics to look over the whole system. In fact, the lognormality emerges rather conspicuously when we examine, as familiar and typical examples of statistical aspects in complex systems, nursing-care period for the aged, populations of prefectures and municipalities, and our body height and weight. One example is shown in Fig.1, in which the rank-plot of GDP in the world is clearly seen to fit with lognormal distribution almost perfectly. Many other examples are found in nature and society. Based on these observations, we would like to discuss the possibility of sociological physics.

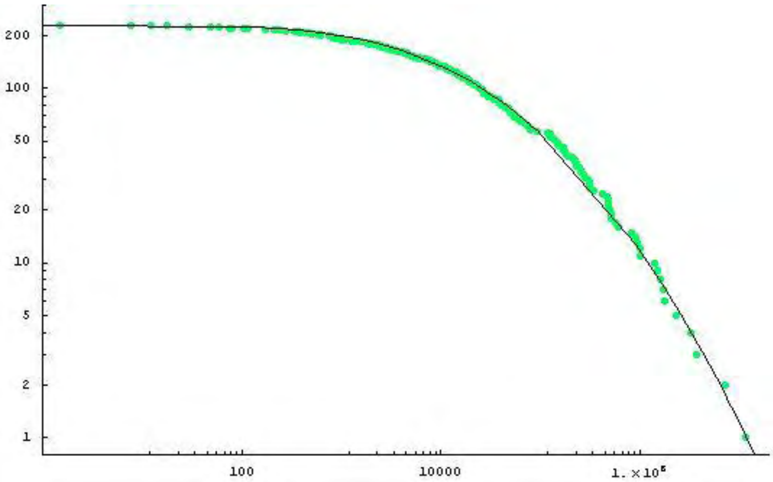

Fig.1 Rank-plot of GDP in the world. The solid line is the best fit with lognormal.

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